Covering Subsets of the Integers and a Result on Digits of Fibonacci Numbers

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A covering system of the integers is a finite system of congruences where each integer satisfies at least one of the congruences. Two questions in covering systems have been of particular interest in the mathematical literature. First is the minimum modulus problem, whether the minimum modulus of a covering system of the integers with distinct moduli can be arbitrarily large, and the second is the odd covering problem, whether a covering system of the integers with distinct moduli can be constructed with all moduli odd. We consider these and similar questions for subsets of the integers, such as the set of prime numbers, the Fibonacci numbers, and numbers that are the sums of two squares. For example, we show that there does exist an odd covering of the integers that are the sums of two squares, and that the minimum modulus problem can be answered in the affirmative for the Fibonacci numbers.

We also define a block of digits in an integer m written in base b as a successive sequence of equal digits of maximal length and define $B(m, b)$ as the number of blocks of m base b. Integers m with $B(m, b) = 1$ are referred to as base b repdigits and have been studied by a number of authors in relationship to recursive sequences, the most famous of which is the Fibonacci sequence. The Fibonacci numbers F_n are defined by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. In particular, Florian Luca was able to show that the largest Fibonacci number which is a base 10 repdigit is $F_{10} = 55$. We expand upon this idea for all integer bases $b \geq 2$, and show that $B(F_n, b)$ tends to infinity as n goes to infinity.